

IN THE UNITED STATES PATENT AND TRADEMARK OFFICE

In re Application of:  
LIPSON, Jesse D.  
Application No.: 10/814,726  
Confirmation No.:  
Filed On: March 31, 2004

Examiner: ZIA

Art Unit: 2131

For: PUBLIC KEY CRYPTOGRAPHIC METHODS  
AND SYSTEM

Mail Stop Amendment  
Commissioner for Patents  
P.O. Box 1450  
Alexandria, VA 22313-1450

**REQUEST FOR CONTINUED EXAMINATION AND REPLY TO FINAL OFFICE  
ACTION MAILED AUGUST 4, 2009**

Dear Sir:

This is in response to the Final Office Action mailed August 4, 2009. In the Office Action the Examiner set a shortened statutory period for response, expiring on November 4, 2009. This Response is being submitted within the shortened statutory time period, with a three-month extension of time on February 4, 2010. This response is being submitted with a Petition to Revive, under 37 CFR 1.137 (a), and the required fees. Applicant respectfully requests the Examiner to reconsider the rejections made in the Office Action in view of the Amendments and Remarks made herein.

**Amendments to the Claims** begin on page 2 of this paper.

**Remarks/Arguments** begin on page 13 of this paper.

## CLAIMS

Please amend the claims as follows:

1. (currently amended) A system for encrypting/decrypting messages, comprising:  
  
a public key cryptosystem further comprising a computer operable for generating keys for use with messages that have been encrypted and/or decrypted wherein the public key cryptosystem having a predetermined number of prime factors used for the generation of a modulus N and an exponent e; wherein the modulus N is not a squareful number;  
  
wherein a proper subset of the prime factors of the modulus N composed of less than all of the distinct prime factors, along with the exponent e, are required to decrypt messages that are encrypted using the public exponent e and the public modulus N, where e and N are calculated using RSA methods, and encryption occurs using RSA methods.
2. (currently amended) A method for encrypting/decrypting messages comprising the steps of:  
  
providing a public key cryptosystem including a computer operable to generate at least one key for encrypting/decrypting at least one message, the public key cryptosystem having a predetermined number of distinct prime factors used for the generation of a modulus N and an exponent e; wherein the modulus N is not a squareful number;  
  
wherein a proper subset of the prime factors of the modulus N composed of less than all of the distinct prime factors are required to decrypt messages that are encrypted using the public exponent e and the public modulus N, where e and N are calculated using RSA methods, and encryption of the message occurs using RSA methods.
3. (currently amended) A method for encrypting/decrypting messages comprising the steps of:

Encrypting on a computer a plaintext message  $M$  into a ciphertext message  $C$  using any method that produces a value equivalent to  $C = M^e \bmod N$ , where  $0 \leq M < N_d$ , such that the ciphertext  $C$  can be decrypted into the plaintext message  $M$  using only  $e$  and the prime factors of  $N_d$

$N$  being the product of all of the numbers in the set  $S$ ;

$N$  is not a squareful number;

$S$  being a set of at least two distinct prime numbers,  $p_1 \dots p_k$ , where  $k$  is an integer greater than 1;

$e$  being a number;

$S_d$  being a proper subset of  $S$  composed of less than all of the distinct prime factors in set  $S$ ;

$N_d$  being the product of all of the numbers in the set  $S_d$ .

4. (original) The method of claim 3, wherein the step of generating the exponent  $e$  includes calculating the exponent  $e$  as a number that is relatively prime to the product of each distinct prime factor of  $N$  minus 1,  $(N_1 - 1) * \dots * (N_j - 1)$  for distinct prime factors of  $N$  1 to  $j$ , where  $j$  is the number of distinct prime factors in  $N$ , or choosing the exponent  $e$  as a small prime number.
5. (currently amended) A method for decrypting encrypted messages comprising the steps of:

determining if a derived modulus  $N_d$  is a squarefree number, and if so,

decrypting on a computer ciphertext  $C$  into message  $M$  wherein message  $M$  was originally an encrypted message that is transformed into electronic, decrypted message  $M$  using any method that produces a value equivalent to  $M = C^d \bmod N_d$ , where  $d$  is generated using the following steps:

calculating the number  $Z_d$  as the product of each prime factor of  $N_d$  minus 1,  $(N_{d1} - 1) * \dots * (N_{dj} - 1)$  for distinct prime factors of  $N_d$  1 to j, where j is the number of distinct prime factors in  $N_d$ ;

generating the exponent d such that the following relationship is satisfied:  $e * d = 1 \bmod Z_d$ .

6. (original) The method according to claim 5, further including the step of:  
directly calculating  $M = C^d \bmod N_d$ .

7. (original) The method according to claim 5, further including the steps of:

calculating separate decryption exponents  $d_{nd1} \dots d_{ndj}$  for all prime factors of  $N_d$  1 to j, where j is the number of prime factors in  $N_d$  so that the following relationship is satisfied for each member of  $N_d$ :  $e * d_{ndi} = 1 \bmod (N_{di} - 1)$ ; and  
performing decryptions of the form  $M_i = C^{d_{ndi}} \bmod N_{di}$  for all prime factors of  $N_d$  from 1 to j, where j is the number of prime factors in  $N_d$ , and then using the values of each  $M_i$  and  $N_{di}$  to reconstruct M.

8. (original) The method of claim 7, wherein the values of each  $M_i$  and  $N_{di}$  restore the plaintext message M using the Chinese Remainder Theorem and/or Garner's algorithm.

9. (cancelled)

10. (cancelled)

11. (cancelled)

12. (currently amended) A public key cryptosystem where messages are decrypted on a computer using a set of prime numbers S and the public exponent e, and messages are encrypted using a squarefree modulus  $N_p$  that is calculated as the product of a set of distinct numbers that is

a proper superset of S composed of distinct numbers, and encryption occurs with standard RSA methods using the public exponent  $e$  and the modulus  $N_p$ .

13. (currently amended) A method for encrypting/decrypting messages, comprising the steps of:

Encrypting on a computer a plaintext message  $M$  into a ciphertext message  $C$  using any method that produces a value equivalent to  $C = M^e \bmod N_p$ , where  $0 \leq M < N$ , such that the ciphertext  $C$  can be decrypted into the plaintext message  $M$  using  $e$  and the distinct prime factors of  $N$

$N$  being the product of all of the numbers in the set  $S$ ;

$N$  is not a squareful number;

$S$  being a set of at least one prime number,  $p_1 \dots p_k$ , where  $k$  is an integer greater than 0;

$S_p$  being a proper superset of  $S$  composed of distinct prime numbers;

$N_p$  being the product of all of the numbers in the set  $S_p$ ;

$e$  being a number.

14. (original) The method of claim 13, wherein the step of generating the exponent  $e$  includes calculating the exponent  $e$  as a number that is relatively prime to the product of each distinct prime factor of  $N_p$  minus 1,  $(N_{p1} - 1) * \dots * (N_{pj} - 1)$  for distinct prime factors of  $N_p$  1 to  $j$ , where  $j$  is the number of distinct prime factors in  $N_p$ .

15. (original) The method of claim 13, wherein the step of generating the exponent  $e$  includes choosing the exponent  $e$  as a small prime number.

16. (cancelled)

17. (cancelled)

18. (cancelled)

19. (currently amended) A method of decrypting encrypted messages, including the steps of:

Decrypting on a computer the ciphertext message C into the plaintext message M by:

determining if the modulus N is a squarefree number; and if so then,

decrypting ciphertext C into message M using any method that produces a value equivalent to  $M = C^d \bmod N$ , where d is generated using the following steps:

Calculating the number Z as the product of each prime factor of N minus 1,  $(N_1 - 1) * \dots * (N_j - 1)$  for prime factors of N 1 to j, where j is the number of distinct prime factors in N;

then generating the decryption exponent d such that the following relationship is satisfied:  $e * d = 1 \bmod Z$ .

20. (original) The method according to claim 19, further including the step of:

directly calculating  $M = C^d \bmod N$ .

21. (original) The method according to claim 19, further including the steps of:

calculating separate decryption exponents  $d_1 \dots d_j$  for all prime factors of N from 1 to j, where j is the number of prime factors in N so that the following relationship is satisfied for each member of N:  $e * d_i = 1 \bmod (N_i - 1)$ ; and performing decryptions of the form  $M_i = C^{d_i} \bmod N_i$  for all prime factors of N from 1 to j, where j is the number of prime factors in N, and then using the values of each  $M_i$  and  $N_i$  to reconstruct M.

22. (original) The method of claim 21, wherein the values of each  $M_i$  and  $N_i$  reconstruct M using the Chinese Remainder Theorem and/or Garner's algorithm.

23. (currently amended) A method for encrypting/decrypting messages comprising the steps of:

Encrypting on a computer a plaintext message  $M$  into a ciphertext message  $C$  using any method that produces a value equivalent to  $C = M^e \bmod N_p$ , where  $0 \leq M < N$ , such that the ciphertext  $C$  can be decrypted into the plaintext message  $M$  using  $e$  and the prime factors of  $N$ .

$N$  being the product of all of the members of set  $S$ ;

$N$  is not a squareful number;

$S$  being a set of at least two numbers,  $p_1 \dots p_k$  where  $k$  is an integer greater than 1 and all members of  $S$  are equal to  $p_s$ , which is a prime number;

$S_p$  being a superset of  $S$  composed of distinct prime numbers;

$N_p$  being the product of all of the numbers in the set  $S_p$ ;

$e$  being a number.

24. (original) The method of claim 23, wherein the step of generating the exponent  $e$  further includes: Calculating the exponent  $e$  as a number that is relatively prime to the product of all of the distinct prime factors of  $N_p$  minus 1,  $(N_{p1} - 1) * \dots * (N_{pj} - 1)$  for distinct prime factors of  $N_p$  1 to  $j$ , where  $j$  is the number of distinct prime factors in  $N_p$ .

25. (original) The method of claim 23, wherein the step of generating the exponent  $e$  includes choosing the exponent  $e$  as a small prime number.

26. (cancelled)

27. (currently amended) A method for encrypting/decrypting messages, comprising the steps of:

Encrypting on a computer a plaintext message  $M$  into a ciphertext message  $C$  using any method that produces a value equivalent to  $C = M^e \bmod N_p$ , where  $0 \leq M < p$ , such that the ciphertext  $C$  can be decrypted into the plaintext message  $M$  using  $e$  and  $p$

$p$  being a prime number;

S being a set containing only the number p;

$S_p$  being a superset of S consisting of distinct prime numbers;

$N_p$  being the product of all members of the set  $S_p$ ;

$N_p$  is not a squareful number;

e being a number.

28. (original) The method of claim 27, wherein the step of generating the exponent e further includes: Calculating the exponent e as a number that is relatively prime to the product of each distinct prime factor of  $N_p$  minus 1,  $(N_{p1} - 1) * \dots * (N_{pj} - 1)$  for distinct prime factors of  $N_p$  1 to j, where j is the number of distinct prime factors in  $N_p$ .

29. (original) The method of claim 27, wherein the step of generating the exponent e includes choosing the exponent e as a small prime number.

30. (currently amended) A method for decrypting encrypted messages, comprising the steps of:

Decrypting on a computer using any method that produces a value equivalent to  $M = C^d \bmod p$ , where p is a not a squareful number and d is generated using the following step:

Calculating d such that the following equation is satisfied:

$$e * d = 1 \bmod (p - 1).$$

31. (currently amended) A method for establishing cryptographic communications, comprising the steps of:

calculating a composite number N, which is formed from the product of distinct prime numbers  $S, p_1, \dots, p_k$  where  $k \geq 1$ .

and N is not a squareful number;



on a computer Encoding a plaintext message  $M$ , to a ciphertext  $C$ , where  $M$  corresponds to a number representative of a message and  $0 \leq M < S$ ;

generating an exponent  $e$ ;

transforming on the computer said plaintext,  $M$ , into said ciphertext,  $C$ , where  $C$  is developed using any method that produces a value equivalent to  $C = M^e \bmod N$ , such that ciphertext  $C$  can be decrypted into plaintext  $M$  using only  $e$  and  $S$ .

32. (original) The method of claim 31, wherein the step of generating the exponent  $e$  further includes: Calculating the exponent  $e$  as a number that is relatively prime to the product of each distinct prime factor of  $N$  minus 1,  $(N_1 - 1), \dots, (N_j - 1)$  for distinct prime factors of  $N$  1 to  $j$ , where  $j$  is the number of distinct prime factors in  $N$ .

33. (original) The method of claim 31, wherein the step of generating the exponent  $e$  includes choosing the exponent  $e$  as a small prime number.

34. (currently amended) A method for decrypting encrypted messages, comprising the steps of:

decoding on a computer the ciphertext message  $C$  to the plaintext message  $M$ , wherein said decoding comprises the step of: transforming said ciphertext message  $C$  to plaintext  $M$ , using any method that produces a value equivalent to  $M = C^d \bmod S$ , where  $S$  is a not a squareful number and  $d$  is generated using the following step:

generating  $d$  such that  $e*d = 1 \bmod (S - 1)$ .

35. (original) A system for encrypting and decrypting electronic communications including a network of computers and/or computer-type devices, such as personal data assistants (PDAs), mobile phones and other devices, in particular mobile devices capable of communicating on the network; generating at least one private key and at least one public key, wherein the at least one

private key is determined based upon any one of a multiplicity of prime numbers that when multiplied together produce  $N$ , which is the modulus for at least one of the public keys, and wherein the modulus  $N$  is not a squareful number.

36. (currently amended) A method for public key decryption where less than all of the distinct prime factors of a number  $N$  are used to decrypt a ciphertext message  $C$  into plaintext message  $M$ , where encryption occurs on a computer with the public key  $\{e, N\}$  using any method that produces a value equivalent to  $C = M^e \bmod N$  and  $N$  is not a squareful number.

37. (currently amended) A method for public key encryption with a public key  $\{e, N\}$  where a plaintext message  $M$  is encrypted on a computer into a ciphertext message  $C$  using any method that produces a value equivalent to  $C = M^e \bmod (N \cdot X)$ , where  $N$  is the public modulus, wherein  $N$  is not a squareful number; and  $X$  is any integer greater than 1.

38. (currently amended) A method for public key decryption of a message that has been encrypted with the public key  $\{e, N\}$  where a ciphertext message  $C$  is decrypted on a computer into a plaintext message  $M$  using any method that produces a value equivalent to  $M = C^d \bmod N_d$ , where  $N_d$  is the product of less than all of the prime factors of the public modulus  $N$  and  $d$  satisfies the equation  $e \cdot d = 1 \bmod Z$ , where  $Z$  is the product of each of the  $k$  prime factors of  $N_d$  minus 1,  $(p_1 - 1) \cdot \dots \cdot (p_k - 1)$  and wherein the modulus  $N$  is not a squareful number.

39. (currently amended) A method for public key decryption of a message that has been encrypted on a computer using any method that produces a value equivalent to  $C = M^e \bmod N$ , where a ciphertext message  $C$  is decrypted into a plaintext message  $M$  using any method that produces a value equivalent to  $M = C^d \bmod N_d$ , where  $N_d$  is the product of less than all of the prime factors of the public modulus  $N$  and  $d$  satisfies the equation  $e \cdot d = 1 \bmod Z$ , where  $Z$  is the

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product of each of the  $k$  prime factors of  $N_d$  minus 1,  $(p_1 - 1) \dots (p_k - 1)$  and where the modulus  
 $N$  is not a squareful number.

## **REMARKS**

### **Summary of the Office Action**

Claims 1-39 are currently pending and at issue in the present application. Applicant respectfully submits no new matter has been added by this Reply. Claims 1-39 are rejected.

Claims 1-39 are rejected under 35 U.S.C. §102(b) as unpatentable over U.S. Patent 6,396,926 to Takagi et al. ("Takagi"). Applicant request reconsideration of these rejections, in light of the amendments and the below remarks.

### **I. Substantive Rejection Under 35 U.S.C. §102**

Examiner has rejected Claims 1-39 under 102(b) as anticipated by Takagi

35 U.S.C. 102(b) states:

A person shall be entitled to a patent unless—

(b) the invention was patented or described in a printed publication in this or a foreign country or in public use or on sale in this country, more than one year prior to the date of application for patent in the United States.

### **Cited Art – Takagi Patent**

The Examiner indicates that the Takagi reference teaches a method for decrypting a ciphertext obtained from a plaintext using a first and second public key, by applying the Chinese remainder theorem (claim 1). The public key N, or public modulus N, is generated from private keys by **at least squaring** one of the private keys, thus making the public modulus N a **squareful number**. Further, Takagi describes an authentication method for verifying the sender-receiver message (claim 5); a decryption apparatus (claim 9); a cipher communication system, comprised of a sender apparatus for the encryption/decryption key generation processing unit,

and a receiver apparatus containing a calculation processing unit, and a decryption processing unit (claim 10); an authentication message sender apparatus, comprising an encryption/decryption key generation processing unit, an authentication message hashing processing unit, and an authentication encryption processing unit (claim 11).

#### Limitations of Rejected Claims

The independent claims were amended to clarify the present invention and to overcome the rejections of the examiner, for substantive rejections.

#### Applicant's Cryptographic Communication Device

Applicant's invention provides systems and methods for encryption of messages using a public and private key cryptosystem. In a method for secure communication or transmission of electronic matter, according to the present invention, wherein the matter is encrypted and decrypted using RSA methods including the steps of: providing at least two private keys and at least one public key for decrypting an electronic communication or transmission, wherein the at private keys are based upon a multiplicity of **distinct** primes that, when multiplied produce a corresponding one of the at least one public key that is **not a squareful number**; encrypting/decrypting the communication or transmission using the at least one public key; and providing at least two private keys capable of decrypting/encrypting the communication or transmission using a single one of any of the at least two private keys. There is structure provided in the amended claims, including a computer and/or computer-type device capable of communicating on a network (supported by originally filed specification, including page 19) to address the claims rejections under 101 and 112.

#### Analysis of Cited Art to Rejected Claims

Takagi describes a method of encryption that generates a **squareful** modulus as the product of a series of **distinct** prime factors wherein one of the prime factors is raised to an exponent k, thus generating a product from **non-distinct** prime factors. Takagi then teaches the decryption of the encoded message using all of the distinct prime factors. The present invention, instead, teaches the generation of a **squarefree** modulus and the decryption of the resulting encoded message using **less than all** of the **distinct** prime factors. The claims have been amended to distinguish over the prior art.

#### Claims

In order for a reference to act as a §102 bar to patentability, the reference must teach each and every element of the claimed invention. *Kalman v. Kimberly-Clark Corp.*, 713 F.2d 760, 771 (Fed. Cir. 1983).

The present invention systems and methods for encrypting/decrypting messages operable on a computer system, computer-type device and/or computer network provide the activity on a machine and/or product, including the steps of: providing a public key cryptosystem including a computer operable to generate at least one key for encrypting/decrypting at least one message, the public key cryptosystem having a predetermined number of **distinct** prime factors used for the generation of a **squarefree** modulus N and an exponent e; wherein a proper subset of **less than all** of the **distinct** prime factors of the modulus N are used to decrypt messages that are encrypted using the public exponent e and the public modulus N, where e and N are calculated using RSA methods, and encryption of the message occurs using RSA methods. These steps and others in the independent claims, now amended, are not included in the Takagi patent.


Without the required teaching of each element as set forth in the claims, it is improper for the Examiner to continue such rejections under §102(e). Therefore, Applicants respectfully request withdrawal of this rejection as to independent Claims 1, 2, 3, 5, 12, 13, 19, 23, 27, 30, 31, 34, 35, 36, 37, 38 and 39. Further, because remaining claims depend from one of these independent claims, adding additional limitations to each, the rejection of these dependent claims under § 102(e) should also be withdrawn. Applicant respectfully requests that the rejections be withdrawn as to Claims 1-39 and those claims allowed.

## CONCLUSION

The Office Action of August 4, 2009, has substantively rejected Claims 1-39 under 35 U.S.C. §102 as unpatentable over Takagi. The amendments and remarks of Applicant address these rejections. Accordingly, Applicant believes the claims are in condition for allowance. Reconsideration of the pending objections and rejections is respectfully requested, and a notice of allowance is respectfully sought. If any issues remain outstanding, incident to the allowance of the application, Examiner Zia is respectfully requested to contact the undersigned attorney at (919) 268-4236 or via email at [jinan@trianglepatents.com](mailto:jinan@trianglepatents.com) to discuss the resolution of such issues, in order that prosecution of the application may be concluded favorably to the applicant, consistent with the applicant's making of a substantial advance in the art and particularly pointing out and distinctly claiming the subject matter that the applicant regards as the invention.

This Office Action response is being submitted on February 4, 2010 via EFS-Web to USPTO with a Petition to Revive under 37 CFR 1.137(a), along with the Petition fees, and the Request for Continued Examination fee.

Respectfully submitted,



Jinan Glasgow, Reg. No. 42585

Triangle Patents, PLLC

PO Box 28539

Raleigh, NC 27611-8539

919-268-4236 phone